Klein Tunneling

PHYS 503 Physics Colloquium Fall 2008

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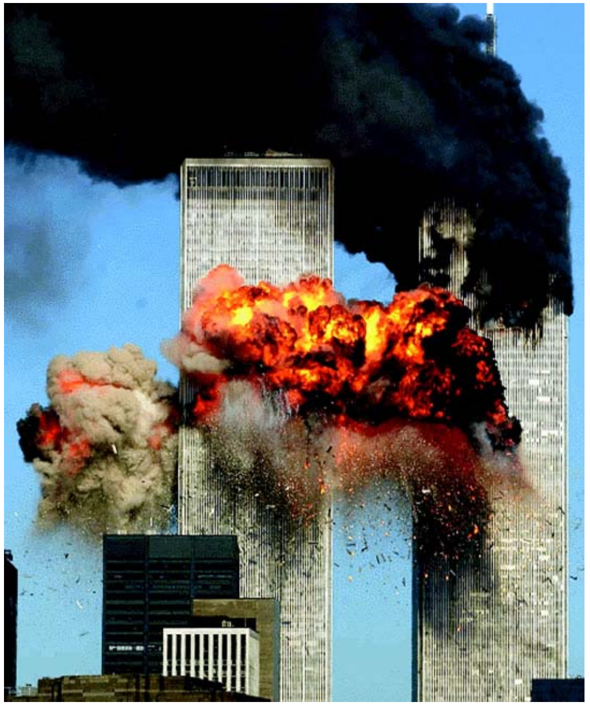

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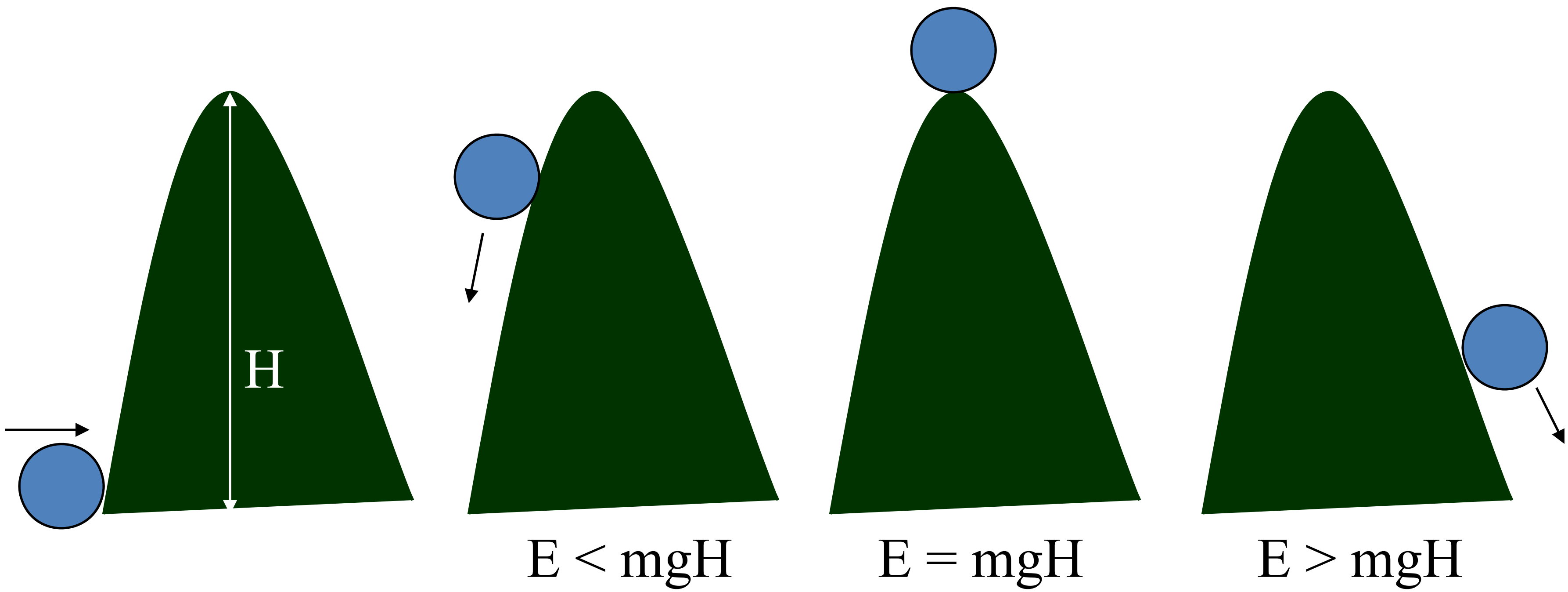
Outline

- Classical picture
- Tunneling
- Klein Tunneling
- Bipolar junctions with graphene
- Applications

Classical Picture



Kinetic Energy = E
Mass of the ball = m

Tunneling

- Transmission of a particle through a potential barrier higher than its kinetic energy (V>E).
- It violates the principles of classical mechanics.
- It is a **quantum** effect.

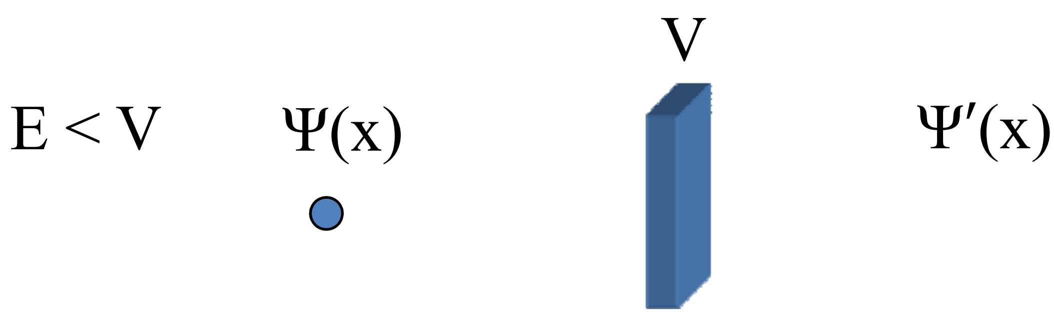
Quantum tunneling effect

- On the quantum scale, objects exhibit ***wave-like*** characteristics.
- Quanta moving against a potential hill can be described by their wave function.
- The wave function represents the ***probability amplitude*** of finding the object in a particular location.

Quantum tunneling effect

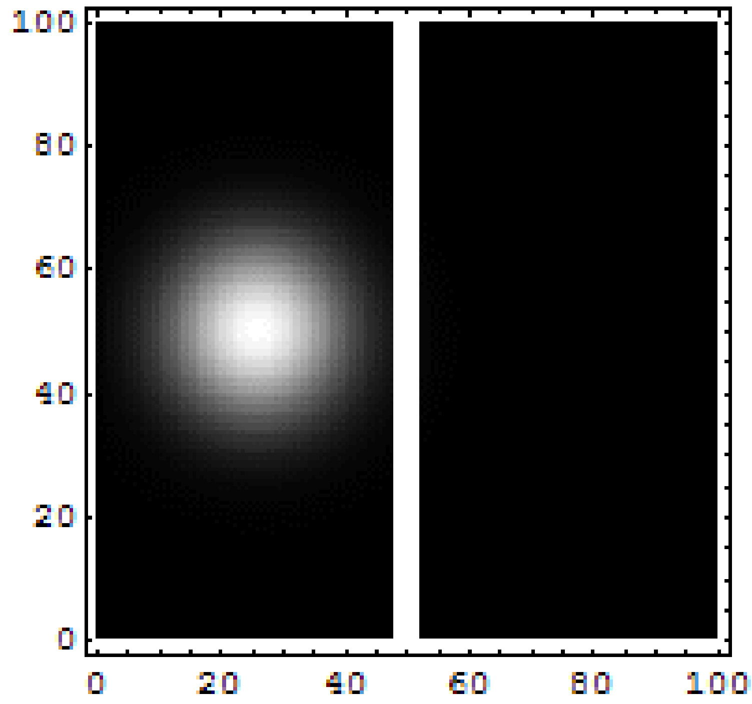
- If this wave-function describes the object as being on the other side of the potential hill, then there is a probability that the object has moved through the potential hill.

- This transmission of the object through the potential hill is termed as ***tunneling***.



Tunneling = Transmission through the potential barrier

Tunneling



Reflection ⟶ Interference fringes

Transmission ⟶ Tunneling

Source: http://en.wikipedia.org/wiki/Quantum_tunneling

Klein Tunneling

- In quantum mechanics, an electron can tunnel from the conduction into the valence band.

- Such tunneling from an electron-like to hole-like state is called as interband tunneling or ***Klein*** tunneling.

- Here, electron avoids backscattering

Tunneling in Graphene

- In graphene, the massless carriers behave differently than ordinary massive carriers in the presence of an electric field.

- Here, electrons avoid backscattering because the carrier velocity is ***independent*** of the energy.

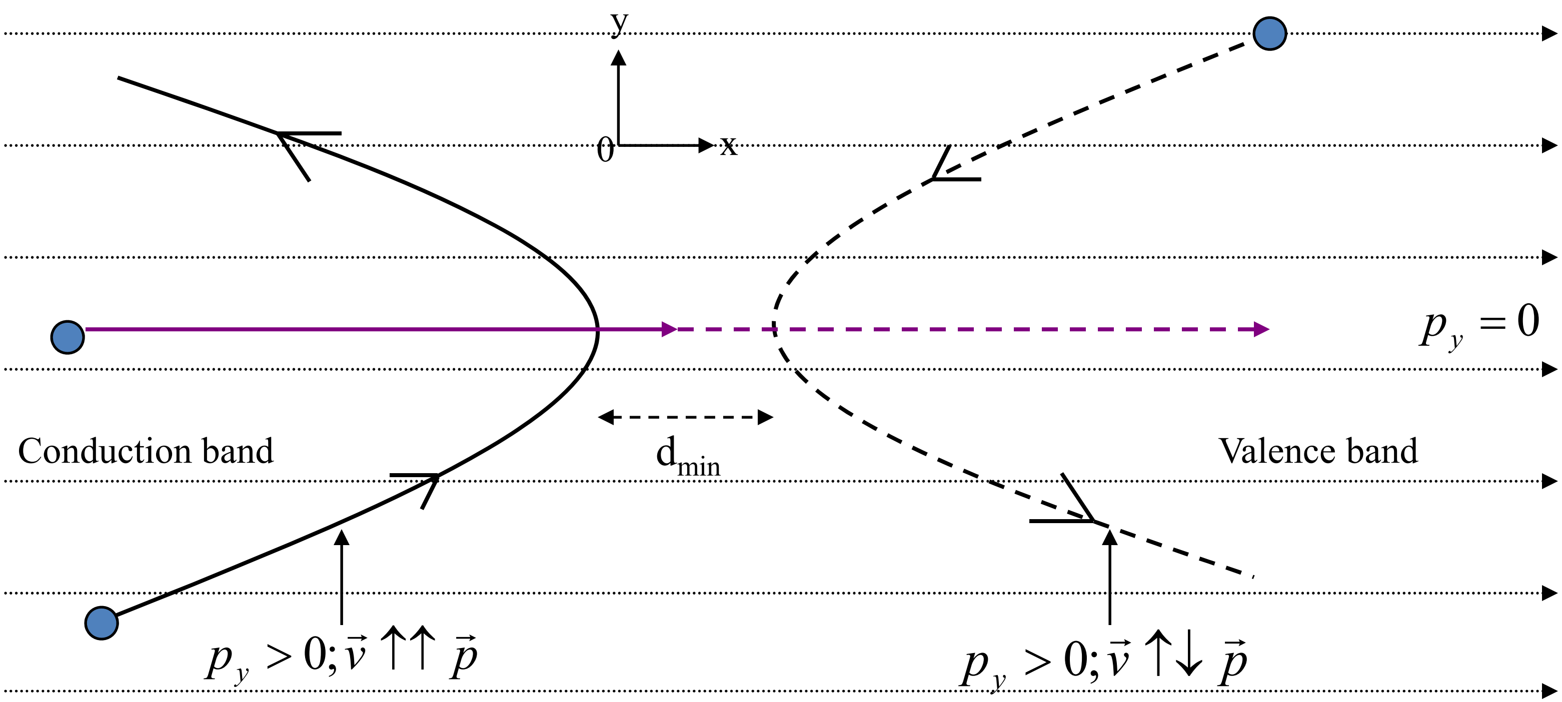
- The absence of backscattering is responsible for the high conductivity in carbon nanotubes (Ando et al, 1998).

Absence of backscattering

Let's consider a linear electrostatic potential

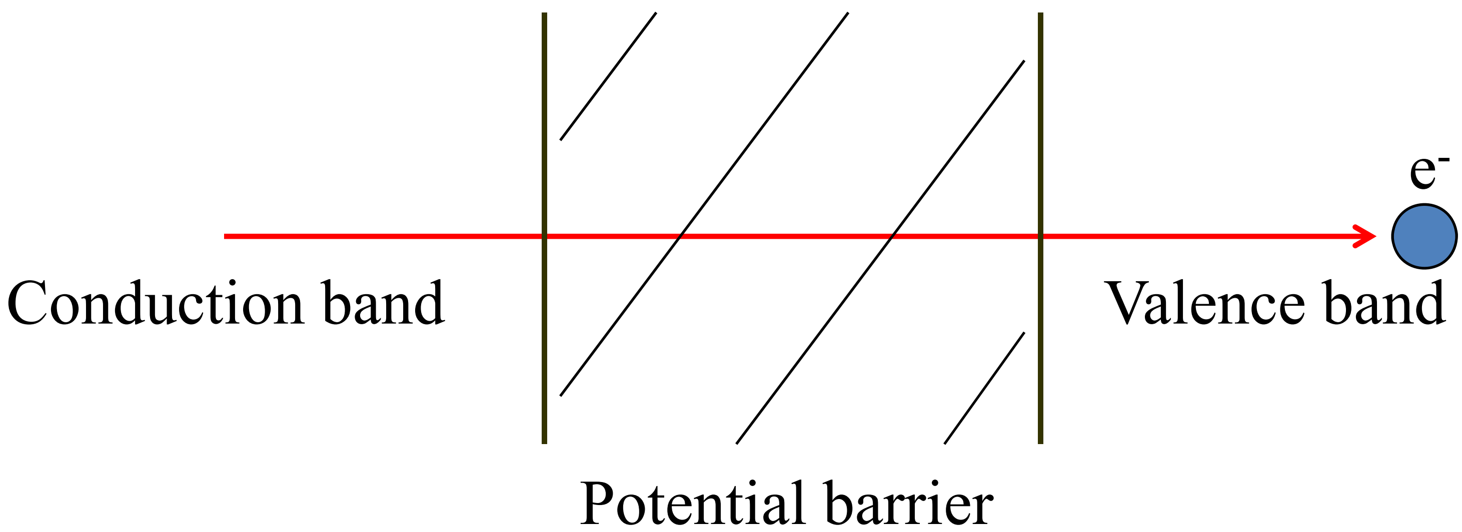
$$U(x) = Fx$$

Electron trajectories will be like:

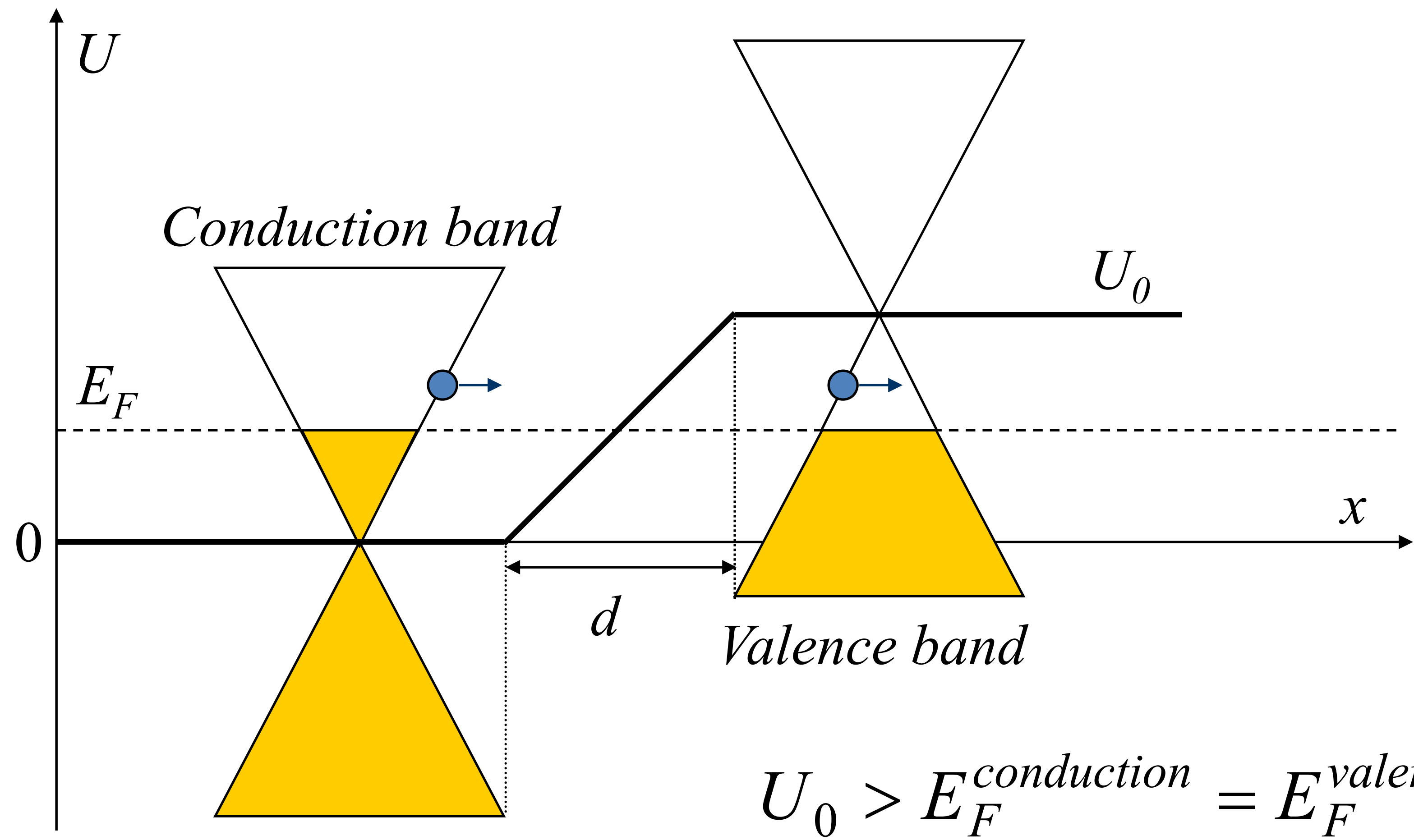


Absence of backscattering

- For $\mathbf{p_y = 0}$, ***no*** backscattering.
- The electron is able to propagate through an infinitely high potential barrier because it makes a *transition* from the conduction band to the valence band.



Band structure



$$U_0 > E_F^{conduction} = E_F^{valence}$$

Absence of backscattering

- In this transition from conduction band to valence band, its dynamics changes from electron-like to hole-like.

- The equation of motion is thus,

$$\frac{d\vec{r}}{dt} \equiv \frac{\partial E}{\partial \vec{p}} = \frac{v^2 \vec{p}}{E - U}$$

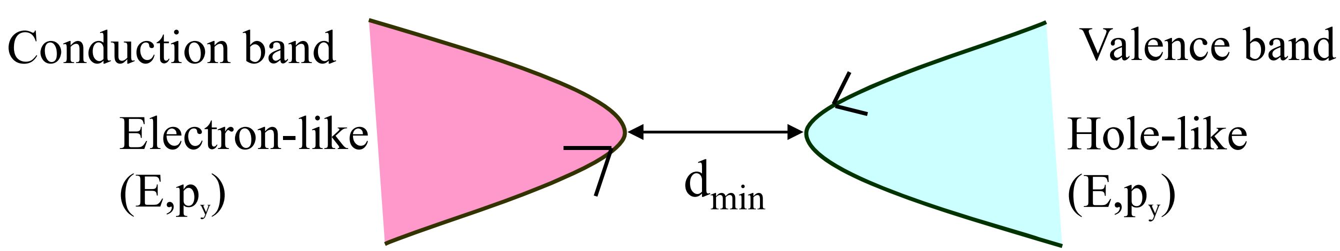
at energy E with $v^2 |\vec{p}|^2 = (E-U)^2$

- It shows that $\vec{v} \uparrow\uparrow \vec{p}$ in the conduction band (U < E) and $\vec{v} \uparrow\downarrow \vec{p}$ in the valence band (U > E).

Klein tunneling

- States with $\vec{v} \uparrow\uparrow \vec{p}$ are called electron-like.
- States with $\vec{v} \uparrow\downarrow \vec{p}$ are called hole-like.
- Pairs of electron-like and hole-like trajectories at the same E and p_y have turning points at:

$$d_{\min} = \frac{2v\,|\,p_y\,|}{F}$$



Klein tunneling

$$d_{\min} = \frac{2v\,|\,p_y\,|}{F}$$

- The tunneling probability: exponential dependence on $d_{\min}$.

$$T(p_y) = \exp\left(\frac{-\pi\,|\,p_y\,|\,d_{\min}}{2\hbar}\right) = \exp\left(\frac{-\pi v {p_y}^2}{\hbar F}\right)$$

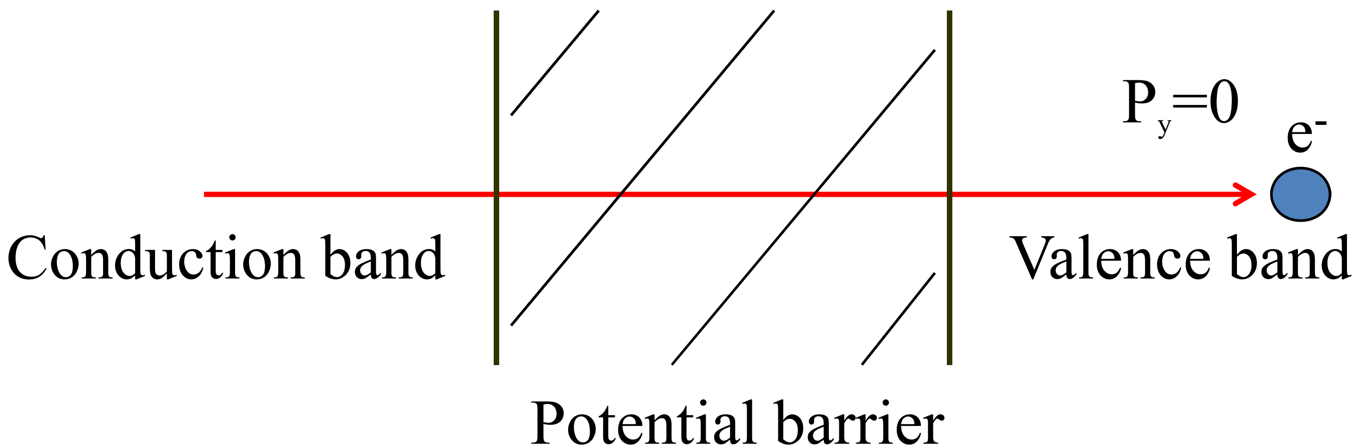
Condition: p_x^{in} at $x \to -\infty$ and p_x^{out} at $x \to \infty$ is sufficiently large :

$$|\,p_x^{in}\,|, |\,p_x^{out}\,| >> |\,p_y\,|, \sqrt{\frac{\hbar F}{v}}$$

Transmission resonance

- It occurs when a p-n junction and an n-p junction form a p-n-p or n-p-n junction.

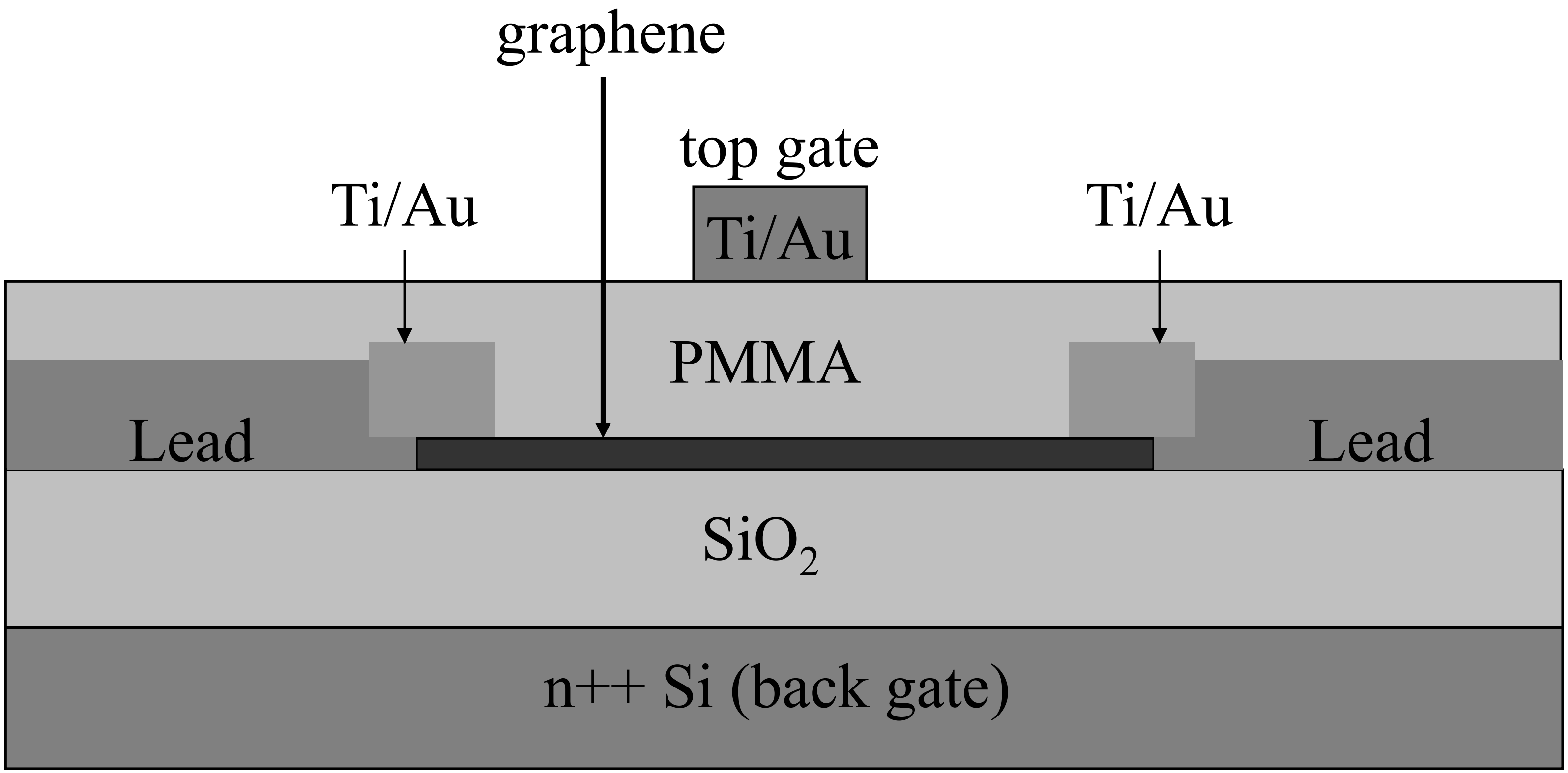
- At p_y=0, T(p_y)=1 (unit transmission): No transmission resonance at normal incidence.



No transmission resonance

Bipolar junctions

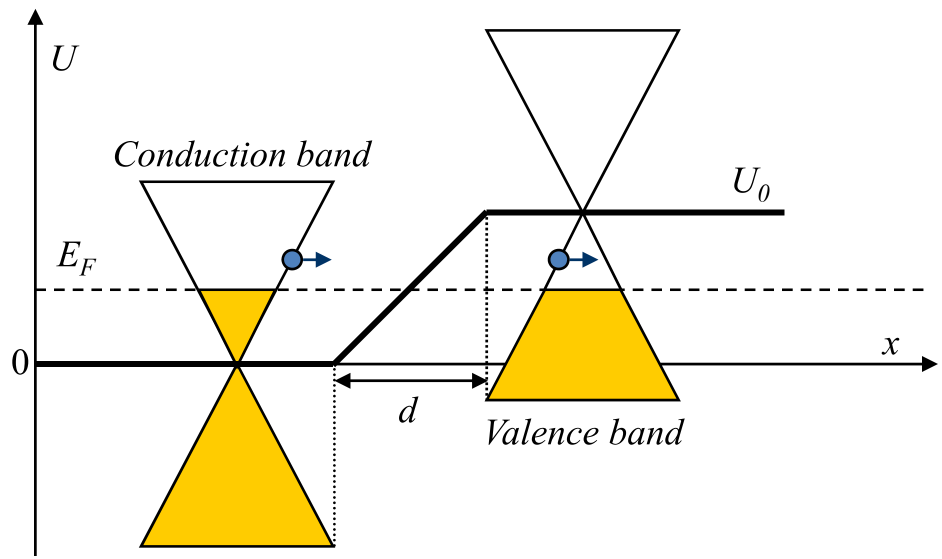
- Electrical conductance through the interface between p-doped and n-doped graphene: **Klein tunneling**.



Bipolar junctions

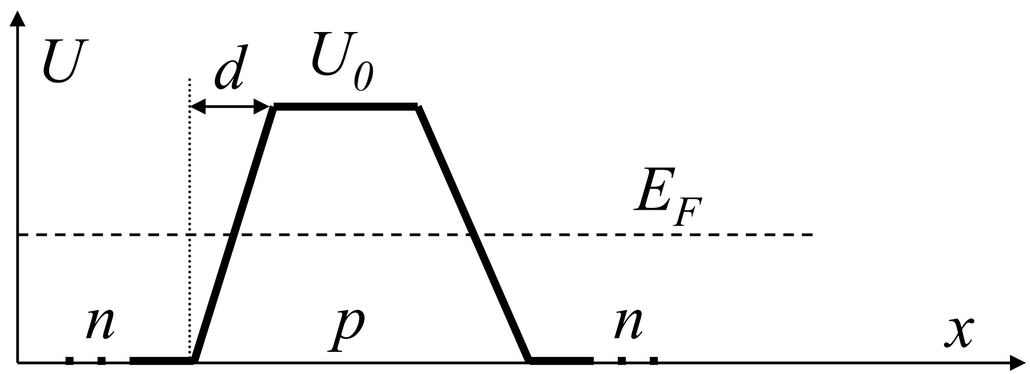
- Top gate: Electrostatic potential barrier
- Fermi level lies

In the *Valence band* inside the barrier (*p*-doped region)	In the *Conduction band* outside the barrier (*n*-doped region)



Bipolar junctions

- Carrier density $n_{carrier}$ is the same in the n and p regions when the Fermi energy is half the barrier height U_0.



Fermi momenta in both the n and p regions are given by:

$$p_F \equiv \hbar k_F = \frac{U_0}{2v} = \frac{Fd}{2v}$$

$d \approx 80$ nm[†]

[†] Measured by Huard et al (2007) for their device.

Bipolar junctions

- The Fermi wave vector $(k_F = \sqrt{\pi n_{carrier}})$ for typical carrier densities of $n_{carrier} \geq 10^{12}\,\mathrm{cm}^{-2}$ is > 10^{-1} nm^{-1}.

- Under these conditions $k_F d > 1$, *p-n* and *n-p* junctions are smooth on the Fermi wavelength.

- The tunneling probability expression can be used.

$$T(p_y) = \exp\left(\frac{-\pi\,|\,p_y\,|\,d_{\min}}{2\hbar}\right) = \exp\left(\frac{-\pi v p_y{}^2}{\hbar F}\right)$$

Bipolar junctions

- The conductance $G_{p\text{-}n}$ of a *p-n* interface can be solved by integration of tunneling probability over the transverse momenta

- The result of integration [†]:

$$G_{p-n} = \frac{4e^2}{h}\frac{W}{2\pi\hbar}\int_{-\infty}^{\infty} dp_y T(p_y) = \frac{4e^2}{h}\frac{W}{2\pi}\sqrt{\frac{F}{\hbar v}}$$

where W is the transverse dimension of the interface.

† Cheianov and Fal'ko, 2006

Applications of tunneling

- Atomic clock
- Scanning Tunneling Microscope
- Tunneling diode
- Tunneling transistor

Questions ?

Who got the Nobel prize (1973) in Physics for his pioneering work on electron tunneling in solids?

Dr. Leo Esaki (b. 1925, Osaka, Japan)

Thanks !!